A Remark On the FRTS realization and Drinfeld Realization of Quantum Affine Superalgebra $U_q(o\hat{s}p(1,2))$

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Abstract

In this paper, we present the hidden symmetry behind the Faddeev-Reshetikhin-Takhtajan-Semenov-Tian-Shansky realization of quantum affine superalgebras $U_q(o\hat{s}p(1,2))$ and add the q-Serre relation to the Drinfeld realization of $U_q(o\hat{s}p(1,2))$ [8] derived from the FRTS realization.

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1 Introduction

For quantum affine algebras, there are two types of loop realizations: 1) the FRTS realization and 2) the Drinfeld realization. For the case of $U_q(\hat{gl}(n))$ [4], the connection of these two realizations are established via the Gauss decomposition of L-operators. This method is recently used to derive Drinfeld realization for quantum affine superalgebras. [8] [11] [2].

Though for the case of $U_q(o\hat{s}p(1,2))$, such a connection was established [8], there are two important aspects of the theory that are still needed to be addressed.

The first aspect is the hidden symmetry of FRTS realization, namely in the FRTS realization, there is a hidden symmetry of the generating L-operators implied in the hidden symmetry of the R-matrix used in the FRTS realization. This hidden symmetry implies that the Drinfeld realization is a quotient of the FRTS realization.

Another aspect of the theory is that of q-Serre relation. Let $E(z)$ $F(z)$ and $H(z)$ be the generating current operators of the affine Lie superalgebra $o\hat{s}p(1,2)$. We know that the defining relations include the Serre relation:

$$E(z)(\{E(w), E(x)\}) = (\{E(w), E(x)\})E(z),$$

$$F(z)(\{F(w), F(x)\}) = (\{F(w), F(x)\})F(z),$$

which clearly is not implied by the relation:

$$(z-w)E(z)E(w) = (w-z)E(w)E(z),$$

$$(z-w)F(z)F(w) = (w-z)F(w)F(z).$$

In this aspect, the Drinfeld realization of $U_q(o\hat{s}p(1,2))$ in [8] is incomplete in the sense that the Serre relation is missing. With the help of the hidden symmetry, we derive a q-Serre relation for the generating current operators of $U_q(o\hat{s}p(1,2))$.

This paper is organized as the following: in Section 2, we recall the main results about $U_q(o\hat{s}p(1,2))$ in [8]; and we present our main results in Section 3.

2 The FRTS realization and the Drinfeld realization of $U_q(o\hat{s}p(1,2))$.

In this section, we will recall the main results and the notation in [8]. The FRTS realization of affine superalgebras starts with a super R-matrix, $R(z) \in End(V \otimes V)$, where V is

a $\mathbf{Z}_2$ graded vector space, and $R(z)$ satisfying the condition $R(z)_{\alpha\beta,\alpha'\beta'} \neq 0$ only when $[\alpha'] + [\beta'] + [\alpha] + [\beta] = 0$ mod2, and the $\mathbf{Z}_2$ graded Yang-Baxter equation (YBE)

$$R_{12}(z)R_{13}(zw)R_{23}(w) = R_{23}(w)R_{13}(zw)R_{12}(z). \tag{2.1}$$

The multiplication for the tensor product is defined for homogeneous elements a, b, a', b' by

$$(a \otimes b)(a' \otimes b') = (-1)^{[b][a']}\,(aa' \otimes bb'), \tag{2.2}$$

where $[a] \in \mathbf{Z}_2$ denotes the grading of the element a. The FRTS realization of quantum affine superalgebras is given as the following.

Definition 1 *: Let $R(\frac{z}{w})$ be a R-matrix satisfying the $\mathbf{Z}_2$ graded YBE (2.1). The FRTS superalgebra $U(R)$ is generated by invertible $L^{\pm}(z)$, satisfying*

$$\begin{aligned} R(\frac{z}{w})L_1^{\pm}(z)L_2^{\pm}(w) &= L_2^{\pm}(w)L_1^{\pm}(z)R(\frac{z}{w}), \\ R(\frac{z_+}{w_-})L_1^{+}(z)L_2^{-}(w) &= L_2^{-}(w)L_1^{+}(z)R(\frac{z_-}{w_+}), \end{aligned} \tag{2.3}$$

where $L_1^{\pm}(z) = L^{\pm}(z) \otimes 1$, $L_2^{\pm}(z) = 1 \otimes L^{\pm}(z)$ and $z_{\pm} = zq^{\pm\frac{c}{2}}$. For the first formula of (2.3), the expansion direction of $R(\frac{z}{w})$ can be chosen in $\frac{z}{w}$ or $\frac{w}{z}$, for the second formula, the expansion direction is in $\frac{z}{w}$.

The algebra $U(R)$ is a graded Hopf algebra: its coproduct is defined by

$$\Delta(L^{\pm}(z) = L^{\pm}(zq^{\pm 1\otimes\frac{c}{2}})\,\dot{\otimes}\,L^{\pm}(zq^{\mp\frac{c}{2}\otimes 1}), \tag{2.4}$$

and its antipode is

$$S(L^{\pm}(z)) = L^{\pm}(z)^{-1}. \tag{2.5}$$

For the case of $U_q(osp(\hat{1},2))$, its FRTS realization is given with the super R-matrix $R(\frac{z}{w}) \in End(V \otimes V)$, where V is the 3-dimensional vector representation of $U_q(osp(1,2))$. In V, we fix a set of basis vectors, v_1, v_2 and v_3, where v_1, v_3 are graded 0 (mod 2) and v_2 is graded 1(mod 2). With this set of basis. the R-matrix is given as:

$$R(\frac{z}{w}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d & 0 & c & 0 & r & 0 & 0 \\ 0 & f & 0 & a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 & e & 0 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a & 0 & b & 0 \\ 0 & 0 & s & 0 & g & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.6}$$

where

$$
\begin{aligned}
a &= \frac{q(z-w)}{zq^2-w}, \qquad b = \frac{w(q^2-1)}{zq^2-w}, \qquad c = \frac{q^{1/2}w(q^2-1)(z-w)}{(zq^2-w)(zq^3-w)}, \\
d &= \frac{q^2(z-w)(zq-w)}{(zq^2-w)(zq^3-w)}, \qquad e = a - \frac{zw(q^2-1)(q^3-1)}{(zq^2-w)(zq^3-w)}, \\
f &= \frac{z(q^2-1)}{zq^2-w}, \qquad g = -\frac{q^{5/2}z(q^2-1)(z-w)}{(zq^2-w)(zq^3-w)}, \\
r &= \frac{w(q^2-1)[q^3z+q(z-w)-w]}{(zq^2-w)(zq^3-w)}, \qquad s = \frac{z(q^2-1)[q^3z+q^2(z-w)-w]}{(zq^2-w)(zq^3-w)}.
\end{aligned} \tag{2.7}
$$

We also have that $R_{21}(\frac{z}{w}) = R(\frac{w}{z})^{-1}$.

To derive the Drinfeld current realization, we need the Gauss decomposition of the L-operators $L^{\pm}(z)$, which is given as

$$
L^{\pm}(z) = \begin{pmatrix} 1 & 0 & 0 \\ e_1^{\pm}(z) & 1 & 0 \\ e_{3,1}^{\pm}(z) & e_2^{\pm}(z) & 1 \end{pmatrix} \begin{pmatrix} k_1^{\pm}(z) & 0 & 0 \\ 0 & k_2^{\pm}(z) & 0 \\ 0 & 0 & k_3^{\pm}(z) \end{pmatrix} \begin{pmatrix} 1 & f_1^{\pm}(z) & f_{1,3}^{\pm}(z) \\ 0 & 1 & f_2^{\pm}(z) \\ 0 & 0 & 1 \end{pmatrix} \tag{2.8}
$$

Let $\{X,Y\} \equiv XY+YX$ denotes an anti-commutator, and

$$
\delta(z) = \sum_{l\in\mathbf{Z}} z^l \tag{2.9}
$$

as a formal series.

Let $X_i^{\pm}(z)$ be defined as

$$
\begin{aligned}
X_i^{+}(z) &= f_{i,i+1}^{+}(z_+) - f_{i,i+1}^{-}(z_-), \\
X_i^{-}(z) &= e_{i+1,i}^{-}(z_+) - e_{i+1,i}^{+}(z_-),
\end{aligned} \tag{2.10}
$$

where $z_\pm = zq^{\pm\frac{c}{2}}$.

Let

$$
X^{\pm}(z) = (q-q^{-1})\left[X_1^{\pm}(z) + X_2^{\pm}(zq)\right]. \tag{2.11}
$$

In [8], the following commutation relations are derived.

Theorem 1

$$
\begin{aligned}
k_1^{\pm}(z)k_1^{\pm}(w) &= k_1^{\pm}(w)k_1^{\pm}(z), \\
k_1^{+}(z)k_1^{-}(w) &= k_1^{-}(w)k_1^{+}(z), \\
k_2^{\pm}(z)k_2^{\pm}(w) &= k_2^{\pm}(w)k_2^{\pm}(z), \\
k_3^{\pm}(z)k_3^{\pm}(w) &= k_3^{\pm}(w)k_3^{\pm}(z), \\
k_3^{+}(z)k_3^{-}(w) &= k_3^{-}(w)k_3^{+}(z),
\end{aligned}
$$

$$
\begin{aligned}
k_1^{\pm}(z)k_2^{\pm}(w) &= k_2^{\pm}(w)k_1^{\pm}(z), \\
\frac{z_{\pm}-w_{\mp}}{z_{\pm}q^2-w_{\mp}}k_1^{\pm}(z)k_2^{\mp}(w) &= \frac{z_{\mp}-w_{\pm}}{z_{\mp}q^2-w_{\pm}}k_2^{\mp}(w)k_1^{\pm}(z), \\
k_1^{\pm}(z)k_3^{\pm}(w)^{-1} &= k_3^{\pm}(w)^{-1}k_1^{\pm}(z), \\
\frac{(z_{\mp}-w_{\pm})(z_{\mp}q-w_{\pm})}{(z_{\mp}q^2-w_{\pm})(z_{\mp}q^3-w_{\pm})}k_1^{\pm}(z)k_3^{\mp}(w)^{-1} &= \frac{(z_{\pm}-w_{\mp})(z_{\pm}q-w_{\mp})}{(z_{\pm}q^2-w_{\mp})(z_{\pm}q^3-w_{\mp})}k_3^{\mp}(w)^{-1}k_1^{\pm}(z), \\
\frac{z_{\pm}-w_{\mp}q}{z_{\pm}q-w_{\mp}}k_2^{\pm}(z)k_2^{\mp}(w) &= \frac{z_{\mp}-w_{\pm}q}{z_{\mp}q-w_{\pm}}k_2^{\mp}(w)k_2^{\pm}(z), \\
k_2^{\pm}(z)^{-1}k_3^{\pm}(w)^{-1} &= k_3^{\pm}(w)^{-1}k_2^{\pm}(z)^{-1}, \\
\frac{z_{\pm}-w_{\mp}}{z_{\pm}q^2-w_{\mp}}k_2^{\pm}(z)^{-1}k_3^{\mp}(w)^{-1} &= \frac{z_{\mp}-w_{\pm}}{z_{\mp}q^2-w_{\pm}}k_3^{\mp}(w)^{-1}k_2^{\pm}(z)^{-1}, \qquad (2.12)
\end{aligned}
$$

$$
\begin{aligned}
k_1^{\pm}(z)X^-(w)k_1^{\pm}(z)^{-1} &= \frac{z_{\pm}q^2-w}{q(z_{\pm}-w)}X^-(w), \\
k_1^{\pm}(z)^{-1}X^+(w)k_1^{\pm}(z) &= \frac{z_{\mp}q^2-w}{q(z_{\mp}-w)}X^+(w), \\
k_2^{\pm}(z)X^-(w)k_2^{\pm}(z)^{-1} &= \frac{(z_{\pm}-wq^2)(z_{\pm}q-w)}{q(z_{\pm}-w)(z_{\pm}-wq)}X^-(w), \\
k_2^{\pm}(z)^{-1}X^+(w)k_2^{\pm}(z) &= \frac{(z_{\mp}-wq^2)(z_{\mp}q-w)}{q(z_{\mp}-w)(z_{\mp}-wq)}X^+(w), \\
k_3^{\pm}(z)X^-(w)k_3^{\pm}(z)^{-1} &= \frac{z_{\pm}-wq^3}{q(z_{\pm}-wq)}X^-(w), \\
k_3^{\pm}(z)^{-1}X^+(w)k_3^{\pm}(z) &= \frac{z_{\mp}-wq^3}{q(z_{\mp}-wq)}X^+(w), \qquad (2.13)
\end{aligned}
$$

$$
\begin{aligned}
\frac{z-wq}{zq-w}X^-(z)X^-(w)+\frac{z-wq^2}{zq^2-w}X^-(w)X^-(z) &= 0, \\
\frac{z-wq^2}{zq^2-w}X^+(z)X^+(w)+\frac{z-wq}{zq-w}X^+(w)X^+(z) &= 0, \qquad (2.14)
\end{aligned}
$$

$$
\begin{aligned}
\{X^-(w),X^+(z)\} = \frac{-1}{q-q^{-1}}\Big[&\delta(\frac{z}{w}q^c)\Big((1+q^{-\frac{1}{2}}-q^{\frac{1}{2}})k_2^+(z_+)k_1^+(z_+)^{-1}-k_3^+(z_+q)k_2^+(z_+q)^{-1}\Big) \\
&-\delta(\frac{z}{w}q^{-c})\Big(k_2^-(w_+)k_1^-(w_+)^{-1}-(1+q^{-\frac{1}{2}}-q^{\frac{1}{2}})k_3^-(w_+q)k_2^-(w_+q)^{-1}\Big)\Big]. \qquad (2.15)
\end{aligned}
$$

In [8], the following definition is proposed:

$$
\begin{aligned}
&\phi_i(z)=k_{i+1}^+(z)k_i^+(z)^{-1}, \\
&\psi_i(z)=k_{i+1}^-(z)k_i^-(z)^{-1}, \quad i=1,2, \\
&\phi(z)=(1+q^{-\frac{1}{2}}-q^{\frac{1}{2}})\phi_1(z)-\phi_2(zq), \\
&\psi(z)=\psi_1(z)-(1+q^{-\frac{1}{2}}-q^{\frac{1}{2}})\psi_2(zq),. \qquad (2.16)
\end{aligned}
$$

Then [8],

Theorem 2 *: $q^{\pm\frac{c}{2}}$, $X^{\pm}(z)$, $\phi(z)$, $\psi(z)$ give the defining relations of $U_q(o\hat{s}p(1,2))$. More precisely, $U_q(o\hat{s}p(1,2))$ is an associative algebra with unit 1 and the Drinfeld generators: $X^{\pm}(z)$, $\phi(z)$ and $\psi(z)$, a central element c and a nonzero complex parameter q. $\phi(z)$ and $\psi(z)$ are invertible. The gradings of the generators are: $[X^{\pm}(z)] = 1$ and $[\phi(z)] = [\psi(z)] = [c] = 0$. The relations are given by*

$$
\begin{aligned}
\phi(z)\phi(w) &= \phi(w)\phi(z), \\
\psi(z)\psi(w) &= \psi(w)\psi(z), \\
\phi(z)\psi(w)\phi(z)^{-1}\psi(w)^{-1} &= \frac{(z_+q - w_-)(z_- - w_+q)(z_+ - w_-q^2)(z_-q^2 - w_+)}{(z_+ - w_-q)(z_-q - w_+)(z_+q^2 - w_-)(z_- - w_+q^2)},
\end{aligned}
$$

$$
\begin{aligned}
\phi(z)X^-(w)\phi(z)^{-1} &= \frac{(z_+ - wq^2)(z_+q - w)}{(z_+q^2 - w)(z_+ - wq)}X^-(w), \\
\phi(z)^{-1}X^+(w)\phi(z) &= \frac{(z_- - wq^2)(z_-q - w)}{(z_-q^2 - w)(z_- - wq)}X^+(w), \\
\psi(z)X^-(w)\psi(z)^{-1} &= \frac{(z_- - wq^2)(z_-q - w)}{(z_-q^2 - w)(z_- - wq)}X^-(w), \\
\psi(z)^{-1}X^+(w)\psi(z) &= \frac{(z_+ - wq^2)(z_+q - w)}{(z_+q^2 - w)(z_+ - wq)}X^+(w),
\end{aligned}
$$

$$
\{X^+(z), X^-(w)\} = \frac{1}{q - q^{-1}}\left[\delta(\frac{w}{z}q^c)\psi(w_+) - \delta(\frac{w}{z}q^{-c})\phi(z_+)\right]. \tag{2.17}
$$

$$
\begin{aligned}
\frac{z - wq}{zq - w}X^-(z)X^-(w) + \frac{z - wq^2}{zq^2 - w}X^-(w)X^-(z) &= 0, \\
\frac{z - wq^2}{zq^2 - w}X^+(z)X^+(w) + \frac{z - wq}{zq - w}X^+(w)X^+(z) &= 0.
\end{aligned} \tag{2.18}
$$

3 The hidden symmetry of FRTS realization and the q-Serre relation of $U_q(o\hat{s}p(1,2))$

As we know, for the classical case, there is a hidden symmetry on the three dimensional representation V of $o\hat{s}p(1,2)$, which comes from fact that V is selfdual. It is , therfore, natural to expect that it is so for the quantum case.

One observation on the results of [8] in the previous section is that there are far too many generating current operators in the FRTS realization than that of the Drinfeld realization. This also clearly indicates that there should be a hidden symmetry of the L-operators, which can help us to resolve this problem.

Let us start with the Heisenberg subalgebra of the FRTS realization. Through calculation (see also the formulas in the previous section), it is not difficult to derive that

Proposition 1 *$k_1^\pm(z)k_3^\pm(zq^3)$ and $k_2^\pm(z)(k_1^\pm(zq^{-1})(k_1^\pm(zq^{-2}))^{-1})^{-1}$ commute with $X_i^\pm(z)$.*

Clearly, from the point view of the theory of universal R-matrix [5][7][8][9], the image of universal R-matrix of $U_q(o\hat{s}p(1,2))$ on $V\otimes V$ actually is $f(z)R(z)$, where $f(z)$ is an analytic function. Therefore, the $L^\pm(z)$ in the definition of $U(R)$ is already different by an multiple of an extra copy of Heisenberg algebra, which is the reason why in [4] we deal with $U_q(\hat{gl}(n))$ not $U_q(\hat{sl}(n))$. Thus, we know that $U(R)$ is actually bigger. The difference comes from the Heisenberg subalgebra generated by $k_1^\pm(z)k_3^\pm(zq^3)$.

We can also derive the following

Proposition 2 *$k_2^\pm(z)(k_1^\pm(z))^{-1}(k_3^\pm(zq)(k_2^\pm(zq))^{-1})^{-1}$ are central in $U(R)$.*

From the point view of the universal R-matrix, we can see that these two central current operators are nothing but identity operators. This shows that either $k_2^\pm(z)(k_1^\pm(z))^{-1}$ or $(k_3^\pm(zq)(k_2^\pm(zq))^{-1})$ as generating operators for the subalgebra generated by $X_i^\pm(z)$.

From now on, we impose the condition that

Condition I

$$k_2^\pm(z)(k_1^\pm(z))^{-1}(k_3^\pm(zq)(k_2^\pm(zq))^{-1})^{-1}=1,$$

on the FRTS realization.

From this and by calculation, we can deduce that

Proposition 3 *Let*

$$Y(z)=X_2^\pm(z)-(\mp q^{\mp\frac{1}{2}}X_1^\pm(zq^{-1}))=\Sigma_{n\in\mathbf{Z}}Y(n)z^{-n}.$$

Let Y be the subalgebra generated by $Y(n), n\in\mathbf{Z}$. Then $YU(R)$ is an double sided ideal in $U(R)$.

We can check that $YU(R)$ has no intersection with the subalgebra generated by $k_i^\pm(z)$, $z=,1,2,3$. If we look at the image of $Y(z)$ on V, we can see that it is actually zero. If we look from the point view of universal R-matrix[9], we should have that

Condition II

$$Y(z)=0, \tag{3.19}$$

which, from now on, we impose on the FRTS realization.

From the above two propositions, we can derive that

Proposition 4 *In the FRTS realization, $X_2^{\pm}(z)$, $k_2^{+}(z)(k_1^{+}(z))^{-1}$ and $k_2^{+}(z)(k_1^{+}(z))^{-1}$, $k_1^{\pm}(z)k_3^{\pm}(zq^3)$ and $k_2^{\pm}(z)(k_1^{\pm}(zq^{-1})(k_1^{\pm}(zq^{-2}))^{-1})^{-1}$ generate the whole algebra.*

This follows the similar calculation for the case $U_q(\hat{sl}(3))$[4] to show $f_{1,3}^{\pm}(z)$ and $e_{3,1}^{\pm}$ are generated by $X_2^{\pm}(z)$.

All the three propositions and two conditions above gives us the hidden symmetry of the L-operators of the FRTS realization of $U_q(o\hat{s}p(1,2))$. On the other hand, we should understand this hidden symmetry comes from the fact that on V, the representation of $U_q(o\hat{s}p(1,2))$ is selfdual. Therefore $R(z)$ has a hidden symmetry like that of $U_q(o(n))$ and $U_q(sp(2n)$ in [6] that basically determines the hidden symmetry of the L-operators[9]. This should automatically lead us to the Condition (I) and (II).

As we explain in the introduction, from the classical theory, we know that the Drinfeld realization given in Theorem 2 in the section above is incomplete in the sense that q-Serre relation is missing. Before we deal with the q-Serre relation, we would like to point out that the relation (2.14) and the similar ones in (2.18) are not completely right. The correct ones are given as following:

Proposition 5

$$\begin{aligned}(zq^2-w)(z-wq)X^-(z)X^-(w)+(zq-w)(z-wq^2)X^-(w)X^-(z) &= 0,\\ (zq-w)(z-wq^2)X^+(z)X^+(w)+(zq^2-w)(z-wq)X^+(w)X^+(z) &= 0. \qquad (3.20)\end{aligned}$$

Here, the point is that the relation given in (2.14) and (2.18) are much stronger than (3.20) in the proposition above, and they actually imply the relations (3.20). However (3.20) do not imply (2.14) and (2.18). The reason comes from the fact that they imply different pole conditions on the products $X^{\pm}(z)X^{\pm}(w)$. Similarly the relations given in [8] about $X_i^{\pm}(z)X_i^{\pm}(w)$ should be corrected correspondingly as in the proposition above.

With rather difficult calculation (similar to that in [4] for the case of $U_q(\hat{sl}(3))$), we can derive that

Proposition 6

$$\frac{(z_3-z_1q^{-1})(z_3-z_1q^3)(z_1-z_2q^2)}{z_3-z_1q}X_i^+(z_3)X_i^+(z_1)X_i^+(z_2)+$$

$$\frac{(z_2-z_1q^2)(z_2-z_1q^{-1})(z_3-z_1q^{-1})(z_3-z_1q^3)}{(z_1-z_2q^{-1})(z_3-z_1q)}X_i^+(z_3)X_i^+(z_2)X_i^+(z_1))-$$

$$\frac{((z_1-z_3q^2)(z_1-z_3q)(z_1q-z_3q^{-1})(z_1-z_2q^2)}{(z_1-z_3)(z_3-z_1q^2)}X_i^+(z_1)X_i^+(z_2)X_i^+(z_3)-$$

$$\frac{((z_1-z_3q^2)(z_1-z_3q)(z_1q-z_3q^{-1})(z_2-z_1q^2)(z_2-z_1q^{-1})}{(z_1-z_3)(z_3-z_1q^2)(z_1-z_2q^{-1})}X_i^+(z_2)X_i^+(z_1)X_i^+(z_3))=0, \tag{3.21}$$

$$\begin{aligned}
&\frac{(z_3-z_1q)(z_3-z_1q^{-3})(z_1-z_2q^{-2})}{z_3-z_1q^{-1}}X_i^-(z_3)X_i^-(z_1)X_i^-(z_2)+\\
&\frac{(z_2-z_1q^{-2})(z_2-z_1q)(z_3-z_1q)(z_3-z_1q^{-3})}{(z_1-z_2q)(z_3-z_1q^{-1})}X_i^-(z_3)X_i^-(z_2)X_i^-(z_1))-\\
&\frac{((z_1-z_3q^{-2})(z_1-z_3q^{-1})(z_1q-z_3q)(z_1-z_2q^{-2})}{(z_1-z_3)(z_3-z_1q^{-2})}X_i^-(z_1)X_i^-(z_2)X_i^-(z_3)-
\end{aligned}$$

$$\frac{((z_1-z_3q^{-2})(z_1-z_3q^{-1})(z_1q-z_3q)(z_2-z_1q^{-2})(z_2-z_1q)}{(z_1-z_3)(z_3-z_1q^{-2})(z_1-z_2q)}X_i^-(z_2)X_i^-(z_1)X_i^-(z_3))=0, \tag{3.22}$$

for $i=1,2,\emptyset$ and where the coefficient functions of the relations above are expanded in the region of the expansion region of the corresponding monomial of the product of $X_i^{\pm}(z_j)$.

We call the two relations above, the q-Serre relations. It is not very difficult to show that this relation will degenerate into the classical Serre relations, but we still do not know how to write a simple formulation like that of the Drinfeld realization of $U_q(\hat{sl}(3))$.

Definition 2 *$U_q(o\hat{s}p(1,2))$ is an $\mathbf{Z}_2$ graded associative algebra generated by c , an central element;*

$$\phi(z)=\sum_{-m\in\mathbf{Z}_{\geq 0}}\phi(-m)z^m;$$
$$\psi(z)=\sum_{m\in\mathbf{Z}_{\geq 0}}\psi(-m)z^m;$$
$$X^{\pm}(z)=\Sigma X(n)z^{-n});$$

where $\phi(z),\psi(z)$ are invertible and $\phi(0)\psi(0)=1=\psi(0)\phi(0)$. The gradings of the generators are: $[\bar{X}^{\pm}(z)]=1$ and $[\phi(z)]=[\psi(z)]=[c]=0$. The relations are given by (2.17), (3.20), (3.22)and (3.23).

Theorem 3 *$U_q(o\hat{s}p(1,2))$ is isomorphic to a quotient of the subalgebra of the FRTS algebra generated by $X^{\pm}(z)$ and $k_3^{\pm}(z)k_2^{\pm}(z)^{-1}$, where the quotient ideal is generated by Condition (I) and (II), and the map is given by*

$$\frac{1}{q-q^{-1}}X_2^{\pm}(zq)\to X^{\pm}(z),$$
$$k_3^+(z)k_2^+(z)^{-1}\to\psi(z),$$
$$k_3^-(z)k_2^-(z)^{-1}\to\phi(z).$$

This theorem also tells us that the FRTS realization U(R) is nothing else but $U_q(o\hat{s}p(1,2))$ tensored by another copy of Heisenberg algebra.

Starting from a completely different point of view, we also come to the same definition of $U_q(o\hat{s}p(1,2))$[3], where the q-Serre relation was derived in a different way.

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